

## Part II

Logarithmic Equations--Equation with one or more logs

ex

$$\log_{100} x = -3/2$$

$$\begin{aligned} 100^{-3/2} &= x \\ 10^{-3} &= x \\ \frac{1}{1000} &= x \end{aligned}$$

ex

$$\log_x 64 = 3$$

$$\begin{aligned} \sqrt[3]{x^3} &= \sqrt[3]{64} \\ x &= 4 \end{aligned}$$

ex

$$\log_x \sqrt{5} = 3/2$$

$$\begin{aligned} x^{3/2} &= \sqrt{5} \\ (x^{3/2})^{2/3} &= (\sqrt{5})^{2/3} \\ x &= 5^{1/3} \\ x &= \sqrt[3]{5} \end{aligned}$$

ex

$$\log_x 3 = -2$$

$$\begin{aligned} (x^{-2})^{1/2} &= (3)^{1/2} \\ x &= \frac{1}{\sqrt{3}} \quad \text{Rationalize} \\ x &= \frac{\sqrt{3}}{3} \end{aligned}$$

ex

$$\log_{4x} 9 = 2$$

$$\begin{aligned} (4x)^2 &= 9 \\ 16x^2 &= 9 \\ x^2 &= \frac{9}{16} \\ x &= \pm \frac{3}{4} \\ x &= \frac{3}{4} \end{aligned}$$

Base  
Must  
be positive

Do:

$$1. \log_x 27 = 3/4$$

$$81 \quad x^{3/4} = 27^{4/4} \quad x = 3^4$$

$$2. \log_{6^x} 3 = 3$$

$$216$$

$$3. \log_x 7 = 1/2$$

$$49$$

$$4. \log_x \sqrt{5} = 1/4$$

$$25$$

## Inequalities

If  $b > 1, x > 0$ and  $\log_b x > y$ , then  $b^y < x$ and  $\log_b x < y$ , then  $b^y > x > 0$ Ex  
 $\log_8 x < 2$ 

$$8^2 > x > 0$$

$$0 < x < 64$$

Ex  
 $\log_5 x > 3$ 

$$5^3 < x$$

$$x > 125$$

$$\log_b x = \log_b y \text{ iff } x = y$$

ex

$$\log_3 (x+2) = \log_3 (2x)$$

$$x+2 = 2x$$

$$2 = x \quad \checkmark$$

ex

$$\log_{10} (3x-4) < \log_{10} (x+6)$$

$$3x-4 < x+6$$

$$2x < 10$$

$$x < 5$$

$$3x-4 > 0$$

$$x > \frac{4}{3}$$

$$x+6 > 0$$

$$x > -6$$

$$\frac{4}{3} < x < 5$$

Cannot take the log of negative or zero

ex

$$\log_4 x^2 = \log_4 (4x-3)$$

$$x^2 = 4x-3$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \quad x = 1$$

{ HW  
p536  
47-61odd, 54, 62